THE DRAWING OF A THIN TUBE THROUGH A CONICAL DIE

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The present paper is concerned with the frictionless drawing of a thin tube through a conical die, assuming that the end section of the tube is free of stress. This problem is related to the problem of deep-drawing, formulated and solved by Hill [1] subject to the yield criterion of Tresca.

Let us consider the problem of the drawing of a thin tube using the usual yield criterion and the corresponding relations between the stress components and the rate of strain components.

The initial distance of a particle in the tube from the symmetry axis will be denoted by r_0 and the end-section radius by a_0 (Fig. 1). The distance of this particle from the axis will be denoted by r when the radius of the end section is a (Fig. 2).

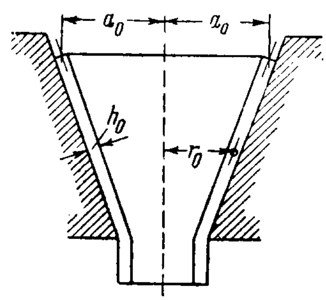

Fig. 1.

The radial velocity v is conveniently measured relative to the radius as the time scale, and the initial radius a_0 can be set equal to unity.

The stress- and strain-rate fields in the conical tube will be determined by the stress components σ_1, σ_2 and by the strain-rate components ϵ_1, ϵ_2 in the meridional and circumferential directions so that

$$\varepsilon_1 = \frac{\partial v}{\partial r}, \qquad \varepsilon_2 = \frac{v}{r}$$

The differential equation of equilibrium of a conical tube of thickness h has the usual form

$$\frac{\partial(h\sigma_1)}{\partial r}+\frac{h(\sigma_1-\sigma_2)}{r}=0 \qquad (1)$$

and the yield criterion is

$$\Phi^2=\sigma_1^2-\sigma_1\sigma_2+\sigma_2^2=\sigma_s^2 \qquad (2)$$

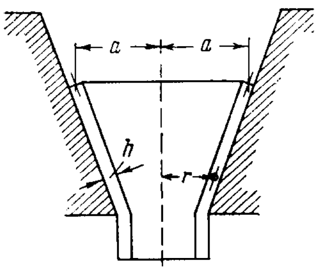

Fig. 2.

The relation between the stress- and strain-rate components is of the form

$$\frac{\varepsilon_1}{\partial\Phi/\partial\sigma_1}=\frac{\varepsilon_2}{\partial\Phi/\partial\sigma_2}, \quad \text{or} \quad \frac{\varepsilon_1}{2\sigma_1-\sigma_2}=\frac{\varepsilon_2}{2\sigma_2-\sigma_1}$$

which leads to

$$\frac{\partial v}{\partial r}=\frac{2\sigma_1-\sigma_2}{2\sigma_2-\sigma_1}\frac{v}{r} \qquad (3)$$

Usually, the condition for the incompressibility of the material is written in the form

$$\frac{1}{h}\left(\frac{\partial h}{\partial a}+v\frac{\partial h}{\partial r}\right)+\frac{\partial v}{\partial r}+\frac{v}{r}=0 \qquad (4)$$

The above system consists of four equations in four unknown functions, namely, σ_1, σ_2, v and h. It belongs to the hyperbolic type and has two families r_0 and a of real characteristics. The family r_0 is given by the differential equations

$$\frac{dv}{v}=\frac{2\sigma_1-\sigma_2}{2\sigma_2-\sigma_1}\frac{dr}{r}, \qquad \frac{dH}{H}=\frac{\sigma_2}{\sigma_1}\frac{dr}{r}-\frac{d\sigma_1}{\sigma_1} \qquad (5)$$

while the family a is given by

$$dr=v\,da, \qquad \frac{dH}{H}=-\frac{2\sigma_1-\sigma_2}{2\sigma_2-\sigma_1}\frac{dr}{r} \qquad (H=rh) \qquad (6)$$

Clearly, the initial conditions are

$$r=r_0, \qquad h=h_0 \qquad \text{for } a=1$$

and the boundary conditions are

$$\sigma_1=0, \qquad v=1 \qquad \text{for } r_0=1$$

Let us express the stress components σ_1 and σ_2 in terms of a new variable ϕ using the substitution

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\}=\frac{2\sigma_s}{\sqrt{3}}\cos\left(\varphi\mp\frac{\pi}{6}\right)$$

so that $\phi = 2\pi/3$ corresponds to $\sigma_1 = 0$.

It is clear that the differential equations given by Equation (5) can now be rewritten in the form

$$\frac{dv}{v} = -\frac{\sin(\varphi + \pi/6)}{\sin(\varphi - \pi/6)}\frac{dr}{r}, \quad \frac{dH}{H} = \frac{\cos(\varphi + \pi/6)}{\cos(\varphi - \pi/6)}\frac{dr}{r} + \tan\left(\varphi - \frac{\pi}{6}\right)d\varphi \tag{7}$$

while the differential equation given by Equation (6) can be rewritten in the form

$$dr = v\,da, \qquad \frac{dH}{H} = \frac{\sin(\varphi + \pi/6)}{\sin(\varphi - \pi/6)}\frac{dr}{r} \tag{8}$$

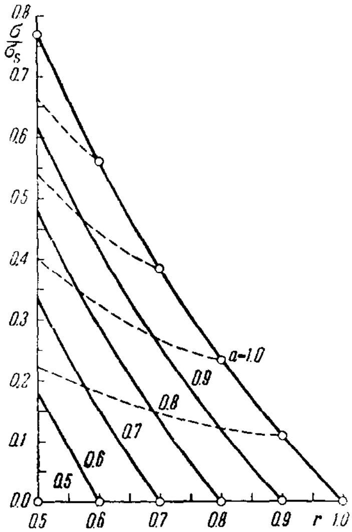

Fig. 3.

Equations (7) and (8), together with the initial and boundary conditions, show that for $a = 1$

$$r^2 = \frac{\sqrt{3}}{2\sin\varphi}\exp\left[-\sqrt{3}\left(\frac{2\pi}{3} - \varphi\right)\right]$$

$$v^2 = \frac{\sqrt{3}}{2\sin\varphi}\exp\left[+\sqrt{3}\left(\frac{2\pi}{3} - \varphi\right)\right]$$

Moreover, for $r_0 = 1$

$$h = \frac{h_0}{\sqrt{a}}$$

Numerical solutions of the differential equations given by Equations (7) and (8), using the method of finite differences, are shown in Figs. 3 and 4. The continuous curves are graphs of $\sigma_1 = \sigma$ and h as functions of r for different values of a between 1.0 and 0.5 (in steps of 0.1). The dashed curves show graphs of σ and h as functions of r for r_0 between 0.5 and 1.0 (in steps of 0.1).

Let us now consider the problem of the drawing of a thin tube using the linearized plasticity conditions and the corresponding relations between stress- and strain-rate components, as put forward by Prager [2].

The differential equation for the equilibrium of a thin tube of thickness h is, as before

$$\frac{\partial(h\sigma_1)}{\partial r} + \frac{h(\sigma_1 - \sigma_2)}{r} = 0 \tag{9}$$

while the yield criterion is

$$\Phi = \mu\sigma_1 - \sigma_2 = \sigma_s \qquad (1/2 \leqslant \mu \leqslant 1) \tag{10}$$

The stress- and the strain-rate components are related by the simple formulas

$$\frac{\varepsilon_1}{\partial\Phi / \partial\sigma_1} = \frac{\varepsilon_2}{\partial\Phi / \partial\sigma_2}, \quad \text{or} \quad \varepsilon_1 + \mu\varepsilon_2 = 0$$

which give

$$\frac{\partial v}{\partial r} + \mu \frac{v}{r} = 0 \qquad (11)$$

The usual condition for the incompressibility of the material is now

$$\frac{1}{h}\left(\frac{\partial h}{\partial a} + v\frac{\partial h}{\partial r}\right) + (1-\mu)\frac{v}{r} = 0 \qquad (12)$$

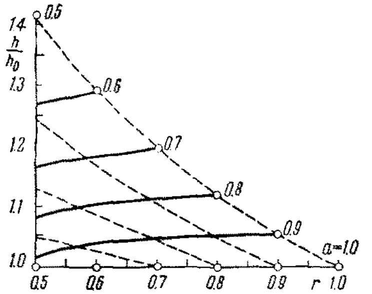

Fig. 4.

The above system of equations consists of four equations in four unknown functions, namely, σ_1, σ_2, v and h. It also belongs to the hyperbolic type and has two families r_0 and a of real characteristics.

The r_0 family is defined by the differential equations

$$\frac{dv}{v} = -\mu\frac{dr}{r}, \qquad \frac{dH}{H} = \frac{\sigma_2}{\sigma_1}\frac{dr}{r} - \frac{d\sigma_1}{\sigma_1} \qquad (13)$$

while the family a is defined by the differential equations

$$dr = v\,da, \qquad \frac{dH}{H} = \mu\frac{dr}{r} \qquad (14)$$

These equations, together with the initial and boundary conditions, enable us to obtain the solution in closed form.

If the constant parameter $\mu \neq 1$, then it is convenient to use the quantities

$$\rho^m = r^{1+\mu}, \qquad \rho_0{}^m = r_0{}^{1+\mu}, \qquad \alpha^m = 1 - a^{1+\mu}$$

$$m = \frac{1+\mu}{1-\mu}$$

remembering that the parameter m lies between 3 and ∞. The variables r, r_0 and a are related by

$$\rho_0{}^m - \rho^m = \alpha^m \quad \text{or} \quad r_0{}^{1+\mu} - r^{1+\mu} = 1 - a^{1+\mu}$$

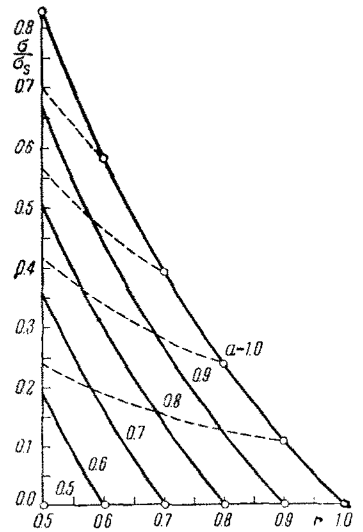

Fig. 5.

The stress component $\sigma_1 = \sigma$ is determined by

$$(1-\mu)\frac{\sigma}{\sigma_s}=\frac{1}{\rho_0}-1+\frac{\alpha^m}{\rho_0}\int_1^{\rho_0}\frac{d\xi}{\alpha^m-\xi^m} \tag{15}$$

and the radial velocity v and thickness h by

$$v=\left(\frac{a}{r}\right)^{\mu}, \qquad h=h_0\left(\frac{r_0}{r}\right)^{1-\mu} \tag{16}$$

The integral which enters into the previous equations for values of

$$\mu=\frac{m-1}{m+1}$$

corresponding to integral values of m, can be expressed in terms of elementary functions. Thus, for example, when $\mu = 1/2$ or $m = 3$, it is clear that

$$\frac{\sigma}{2\sigma_s}=\frac{1}{\rho_0}-1+\frac{\alpha}{\sqrt{3}\rho_0}\left[\tan^{-1}\frac{\sqrt{3}\xi}{\xi+2\alpha}+\frac{1}{\sqrt{3}}\ln\frac{\sqrt{\xi^2+\xi\alpha+\alpha^2}}{\xi-\alpha}\right]_1^{\rho_0}$$

If, on the other hand, $\mu = 1$, then the solution of the problem is particularly simple. The variables r, r_0 and a are related by

$$r_0^2-r^2=1-a^2$$

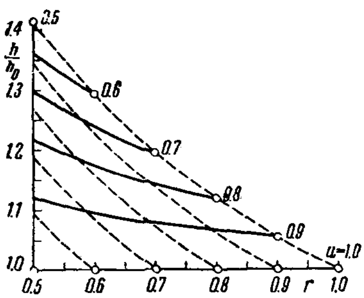

Fig. 6.

The stress component $\sigma_1 = \sigma$ is given by

$$\frac{\sigma}{\sigma_s}=\ln\frac{a}{r}$$

and the radial velocity v and the thickness h are given by

$$v=\frac{a}{r}, \qquad h=h_0$$

Numerical solutions based on Equations (15) and (16) with $\mu = 1/2$ are plotted in Figs. 5 and 6. The continuous curves show $\sigma_1 = \sigma$ and h as functions of r for values of a between 1.0 and 0.5 (in steps of 0.1). The dashed curves show σ and h as functions of r for values of r_0 between 0.5 and 1.0 (also in steps of 0.1). Comparison of σ and h obtained by a numerical solution of the differential equations (7) and (8) by the method of finite differences, with the values of σ and h obtained from Equation (15) and (16), shows a considerable difference between them.

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